Larmor precession and dwell time of a relativistic particle scattered by a rectangular quantum well

2003 J. Phys. A: Math. Gen. 36 6563

(http://iopscience.iop.org/0305-4470/36/23/319)

J. Phys. A: Math. Gen. **36** (2003) 6563–6570 PII: S0305-4470(03)57751-4

Larmor precession and dwell time of a relativistic particle scattered by a rectangular quantum well

Zhi-Jian Li[1,2], **Yi-Hang Nie**[1], **Jun-Jun Liang**[1] **and Jiu-Qing Liang**[1]

[1] Department of Physics, Shanxi University, Taiyuan Shanxi 030006, People's Republic of China
[2] State Key Laboratory of Magnetism, Institute of Physics, Chinese Academy of Sciences, Beijing 100080, People's Republic of China

E-mail: zjli5@yahoo.com

Received 31 December 2002, in final form 19 March 2003
Published 29 May 2003
Online at stacks.iop.org/JPhysA/36/6563

Abstract

The Larmor precession of a relativistic neutral spin-$\frac{1}{2}$ particle in a uniform constant magnetic field confined to the region of a one-dimensional rectangular potential well is investigated. The spin precession serves as a clock to measure the time spent by a quantum particle dwelling at a potential well. With the help of a general spin coherent state it is explicitly shown that the spin precession time is equal to the dwell time in the first-order approximation of the infinitesimal field limit. The comparison of the time in a potential well with that in free space shows apparent superluminality.

PACS numbers: 03.65.Xp, 03.65.Ta

The traversal time of a particle through a quantum potential barrier is a long-standing interesting academic problem. There have been numerous theories about traversal time corresponding to different criteria. Some theories predict that the tunnelling speed is faster than the velocity of light in vacuum, whereas others state that it should be subluminal. So there is no clear consensus on this problem [1–3]. Recently, the experimental reports [4–6] showing apparent superluminality have attracted considerable attention to this subject.

Li and Wang [7] investigated the phase time of a particle scattered by a potential well instead of tunnelling through a barrier. The phase time [8], one of the notions of traversal time, is defined as the sensitivity of the phase of the traversal amplitude to the frequency of the incident particle. They predicted a nonevanescent propagation, even with negative phase shifts. Negative phase shifts lead to propagation with negative group velocities, which means that it appears as if parts of a pulse leave the well before they enter. Because of the analogy of the Schrödinger and Helmholtz equations, this conjecture was confirmed by the experimental data of electromagnetic wave propagation in wave guides [9], where the electromagnetic well was realized by wave guides filled with different dielectrics. It is worth noting that apparent superluminality is distinguishable from true superluminality [10]. Most experimental papers

0305-4470/03/236563+08$30.00

are careful to emphasize their consistency with Maxwell's equations and do not claim the observation of a true superluminal effect, but indeed their results illuminate an interesting and potentially useful effect. As stated in [10], apparent superluminality is extremely general and to be expected.

Most of investigations are focused on nonrelativistic tunnelling and little work has been done towards the study of relativistic tunnelling time. The question 'does the fully relativistic treatment predict apparent superluminal speeds?' has motivated us to investigate the tunnelling time for a particle through a potential barrier in the relativistic regime [11, 12]. We use the spin precession of a relativistic particle in the constant magnetic field as a clock to measure the time spent by the particle penetrating a potential barrier and demonstrate apparent superluminality. The analogous method has been used to investigate the tunnelling time for a nonrelativistic particle through a potential barrier [13, 14]. The precession time, known as Larmor time [15–17], an alternative notation of traversal time, is defined as the precession angular change of the traversal particle divided by the Larmor precession frequency. The relation between the quantum traversal time and the Larmor precession is fully studied in many papers [17–21]. The previous work [17] defines three different Larmor times corresponding to three components of an operator. But with the help of the general spin coherent state we identify a unique Larmor time in the first-order approximation of the infinitesimal field limit and show that this Larmor time exactly equals the dwell time, a physically more significant notion, which measures how long the matter wave remains in the potential barrier regardless of whether the particle is reflected or transmitted [22]. By comparing equation (24) in [11] with equation (2.20*b*) in [17], it is obvious that this Larmor time in the nonrelativistic tunnelling case just corresponds with the Larmor time τ_y in [17]. If we plot the variance of the Larmor time with the width of potential barrier from the expressions of equations (24) and (57) in [11], the independence of the Larmor time from the potential width is also shown, which is just the Hartman effect [23]. Qualitatively, this character is consistent with the result of [24], in which Kential *et al* solve numerically the time-dependent Dirac equation for a quantum wave packet tunnelling through a potential barrier and obtain that the variance of the effective tunnelling speed with increasing barrier width becomes linear when the packet width is larger than the effective width of the barrier.

In [12], in the first-order approximation of the infinitesimal field limit, we present a general proof that the Larmor time equals the dwell time for a relativistic particle penetrating an arbitrary shape potential and this equality includes transmission time and reflection time for a symmetric potential. Therefore, for a symmetric potential, by comparing the Larmor time with potential with that for the particle to penetrate a uniform constant magnetic field but without potential, apparent superluminality can be shown. In this paper, we demonstrate apparent superluminality of a relativistic particle scattering by a rectangular potential well instead of a barrier.

We consider that a relativistic neutral spin-$\frac{1}{2}$ particle with momentum p and mass m in a general spin coherent state impinges on a rectangular potential well U that extends from $-a/2$ to $a/2$. A weak uniform constant magnetic field $\mathbf{B}$, aligned along the z-direction and confined within the potential well region, superimposes the well. The Hamiltonian is seen to be [25]

$$\begin{aligned} H &= c\alpha_1 p_x + \beta mc^2 \qquad |x| > a/2 \\ H &= c\alpha_1 p_x + \beta[(mc^2 - U) - V\Sigma_3] \qquad |x| < a/2 \end{aligned} \tag{1}$$

where $V = \hbar\omega_L/2$ represents the spin–field interaction and $\omega_L = 2\mu B/\hbar$ is the Larmor frequency. μ and $\hbar$ denote the magnetic moment and Planck's constant (divided by 2π)

respectively. β, α_i and Σ_i $(i = 1, 2, 3)$ are expressed as

$$\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} \qquad \Sigma_i = \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} \tag{2}$$

where σ_i are Pauli spin matrices and $S_i = \frac{\hbar}{2}\Sigma_i$ is the spin operator.

We construct the wavefunction with the incoming particle wave polarized along an arbitrary direction satisfying the stationary Dirac–Pauli equation

$$H\psi = E\psi \tag{3}$$

that is

$$\begin{aligned}
\psi_1 &= \frac{1}{\sqrt{1+f_0^2}} \begin{pmatrix} u_1 \\ u_2 \\ f_0 u_2 \\ f_0 u_1 \end{pmatrix} \mathrm{e}^{\frac{\mathrm{i}k_0 x}{\hbar}} + \begin{pmatrix} A_1 \\ A_2 \\ -f_0 A_2 \\ -f_0 A_1 \end{pmatrix} \mathrm{e}^{-\frac{\mathrm{i}k_0 x}{\hbar}} \qquad x < -a/2 \\
\psi_2 &= \begin{pmatrix} B_1\,\mathrm{e}^{\frac{\mathrm{i}k_1 x}{\hbar}} \\ B_2\,\mathrm{e}^{\frac{\mathrm{i}k_2 x}{\hbar}} \\ f_2 B_2\,\mathrm{e}^{\frac{\mathrm{i}k_2 x}{\hbar}} \\ f_1 B_1\,\mathrm{e}^{\frac{\mathrm{i}k_1 x}{\hbar}} \end{pmatrix} + \begin{pmatrix} C_1\,\mathrm{e}^{-\frac{\mathrm{i}k_1 x}{\hbar}} \\ C_2\,\mathrm{e}^{-\frac{\mathrm{i}k_2 x}{\hbar}} \\ -f_2 C_2\,\mathrm{e}^{-\frac{\mathrm{i}k_2 x}{\hbar}} \\ -f_1 C_1\,\mathrm{e}^{-\frac{\mathrm{i}k_1 x}{\hbar}} \end{pmatrix} \qquad -a/2 < x < a/2 \\
\psi_3 &= \begin{pmatrix} D_1 \\ D_2 \\ f_0 D_2 \\ f_0 D_1 \end{pmatrix} \mathrm{e}^{\frac{\mathrm{i}k_0 x}{\hbar}} \qquad x > a/2.
\end{aligned} \tag{4}$$

This leads for energies $E > U - mc^2$ to wave propagation with the real momentum

$$\begin{aligned}
k_0 &= \frac{1}{c}\sqrt{E^2 - (mc^2)^2} \\
k_1 &= \frac{1}{c}\sqrt{(E+V)^2 - (mc^2 - U)^2} \\
k_2 &= \frac{1}{c}\sqrt{(E-V)^2 - (mc^2 - U)^2}
\end{aligned} \tag{5}$$

and

$$\begin{aligned}
f_0 &= \frac{ck_0}{mc^2 + E} \\
f_1 &= \frac{ck_1}{mc^2 - U + E + V} \\
f_2 &= \frac{ck_2}{mc^2 - U + E - V}.
\end{aligned} \tag{6}$$

The incoming wave is assumed to be a normalized spin coherent state which is an eigenstate of the spin operator $\boldsymbol{\sigma} \cdot \mathbf{n}$ where $\mathbf{n} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ denotes the arbitrary unit vector with a polar angle θ and azimuthal angle φ. The two components of the spinor are

$$u_1 = \cos\frac{\theta}{2}\,\mathrm{e}^{-\mathrm{i}\varphi/2} \qquad u_2 = \sin\frac{\theta}{2}\,\mathrm{e}^{\mathrm{i}\varphi/2}. \tag{7}$$

The coefficients A_i, B_i, C_i and D_i $(i = 1, 2)$ in the wavefunction are obtained from boundary conditions $\psi_1(-a/2) = \psi_2(-a/2)$ and $\psi_2(a/2) = \psi_3(a/2)$, namely,

$$\begin{aligned} D_i &= \sqrt{T_i}\, \mathrm{e}^{\mathrm{i}\phi_i}\, \mathrm{e}^{-\frac{\mathrm{i}ak_0}{\hbar}} u_i \\ A_i &= \sqrt{R_i}\, \mathrm{e}^{-\mathrm{i}\frac{\pi}{2}}\, \mathrm{e}^{\mathrm{i}\phi_i}\, \mathrm{e}^{-\frac{\mathrm{i}ak_0}{\hbar}} u_i \\ B_i &= \frac{f_0 + f_i}{2f_i}\, \mathrm{e}^{\frac{\mathrm{i}a(k_0-k_i)}{2\hbar}} D_i \\ C_i &= \frac{-f_0 + f_i}{2f_i}\, \mathrm{e}^{\frac{\mathrm{i}a(k_0+k_i)}{2\hbar}} D_i \end{aligned} \tag{8}$$

where

$$\begin{aligned} T_i &= \frac{4f_0^2 f_i^2}{\left(1+f_0^2\right)\left[\left(f_0^2 - f_i^2\right)^2 \sin^2\left(\frac{ak_i}{\hbar}\right) + 4f_0^2 f_i^2\right]} \\ R_i &= \frac{\left(f_0^2 - f_i^2\right)^2 \sin^2\left(\frac{ak_i}{\hbar}\right)}{\left(1+f_0^2\right)\left[\left(f_0^2 - f_i^2\right)^2 \sin^2\left(\frac{ak_i}{\hbar}\right) + 4f_0^2 f_i^2\right]} \\ \phi_i &= \arctan\left(\frac{f_0^2 + f_i^2}{2f_0 f_i} \tan\frac{ak_i}{\hbar}\right). \end{aligned} \tag{9}$$

For our purpose we consider the first approximation of the infinitesimal field limit,

$$\begin{aligned} k_1 &\simeq k + \frac{E}{c^2k}V \qquad & f_1 &\simeq \frac{k}{\xi} + \frac{1}{c\xi}\left(\frac{E}{ck} - \frac{k}{\xi}\right)V \\ k_2 &\simeq k - \frac{E}{c^2k}V \qquad & f_2 &\simeq \frac{k}{\xi} - \frac{1}{c\xi}\left(\frac{E}{ck} - \frac{k}{\xi}\right)V \end{aligned} \tag{10}$$

where

$$k = \frac{1}{c}\sqrt{E^2 - (mc^2 - U)^2} \qquad \xi \equiv \frac{1}{c}(mc^2 - U + E) \tag{11}$$

is the zero-order approximation. The transmission and reflection probabilities can be expanded as the power series of the small quantity EV/c^2k. The first-order approximation is

$$\begin{aligned} T_1 &= T(k_1) \simeq T(k) - \frac{\partial T}{\partial k}\frac{E}{c^2k}V \qquad & T_2 &= T(k_2) \simeq T(k) + \frac{\partial T}{\partial k}\frac{E}{c^2k}V \\ R_1 &= R(k_1) \simeq R(k) - \frac{\partial R}{\partial k}\frac{E}{c^2k}V \qquad & R_2 &= R(k_2) \simeq R(k) + \frac{\partial R}{\partial k}\frac{E}{c^2k}V. \end{aligned} \tag{12}$$

The expectation values of spin for the transmitted wave are obtained in the infinitesimal field limit as

$$\begin{aligned} \langle S_1\rangle_t &= \left(1+f_0^2\right)\frac{\hbar}{2}T(k)\sin\theta\cos(\phi_2 - \phi_1 + \varphi) \\ \langle S_2\rangle_t &= \frac{\hbar}{2}\left(1-f_0^2\right)T(k)\sin\theta\sin(\phi_2 - \phi_1 + \varphi) \\ \langle S_3\rangle_t &= \frac{\hbar}{2}\left(1-f_0^2\right)\left(T(k)\cos\theta - \frac{\partial T(k)}{\partial k}\frac{E}{ck}V\right). \end{aligned} \tag{13}$$

The reflected part reads

$$\begin{aligned} \langle S_1\rangle_r &= \frac{\hbar}{2}\left(1+f_0^2\right)R(k)\sin\theta\cos(\phi_2 - \phi_1 + \varphi) \\ \langle S_2\rangle_r &= \frac{\hbar}{2}\left(1-f_0^2\right)R(k)\sin\theta\sin(\phi_2 - \phi_1 + \varphi) \\ \langle S_3\rangle_r &= \frac{\hbar}{2}\left(1-f_0^2\right)\left(R(k)\cos\theta - \frac{\partial R}{\partial k}\frac{E}{ck}V\right). \end{aligned} \tag{14}$$

The sum of expectation values of spin components for the reflected and transmitted waves with an infinitesimal magnetic field is

$$\begin{aligned}\langle S_1\rangle &= \frac{\hbar}{2}\sin\theta\cos(\phi_2-\phi_1+\varphi)\\ \langle S_2\rangle &= \frac{\hbar}{2}\frac{1-f_0^2}{1+f_0^2}\sin\theta\sin(\phi_2-\phi_1+\varphi)\\ \langle S_3\rangle &= \frac{\hbar}{2}\frac{1-f_0^2}{1+f_0^2}\cos\theta\end{aligned}\tag{15}$$

which are formally the same as the Larmor precession equation of spin operator **S** in a magnetic field. To see this we solve the Heisenberg equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{S}(t) = \frac{1}{\mathrm{i}\hbar}[\mathbf{S}(t), H_s]\tag{16}$$

with the Hamiltonian

$$H_s = -\tfrac{1}{2}\hbar\omega_L\beta\Sigma_3\tag{17}$$

and the initial wavefunction

$$\psi_i = \frac{1}{\sqrt{1+f_0^2}}\begin{pmatrix}u_1\\ u_2\\ f_0u_2\\ f_0u_1\end{pmatrix}.\tag{18}$$

The expectation values of the spin components at time t are

$$\begin{aligned}\langle S_1(t)\rangle &= \frac{\hbar}{2}\sin\theta\cos(-\omega_L t+\varphi)\\ \langle S_2(t)\rangle &= \frac{\hbar}{2}\frac{1-f_0^2}{1+f_0^2}\sin\theta\sin(-\omega_L t+\varphi)\\ \langle S_3(t)\rangle &= \frac{\hbar}{2}\frac{1-f_0^2}{1+f_0^2}\cos\theta.\end{aligned}\tag{19}$$

Comparing equations (15) and (19), the Larmor tunnelling time τ_L is obviously obtained as

$$\tau_L = \omega_L^{-1}(\phi_1-\phi_2).\tag{20}$$

Using the approximation (10) in equation (9), the Larmor tunnelling time is found to be

$$\tau_L = \frac{f_0}{c^2k}\frac{2ak\xi E\left(k^2+f_0^2\xi^2\right)-\hbar(ck^2-E\xi)\left(k^2-f_0^2\xi^2\right)\sin\left(\frac{2ak}{\hbar}\right)}{4f_0^2\xi^2k^2+\left(k^2-f_0^2\xi^2\right)^2\sin^2\left(\frac{ak}{\hbar}\right)}.\tag{21}$$

The dependence of Larmor time τ_L on the well thickness a is shown in figure 1, where we assume the particle kinetic energy $E_p = E - mc^2 = 0.5mc^2$ and $U = 0.1mc^2, 0.4mc^2, 0.8mc^2, 1.6mc^2$ corresponding to the dash-dot, solid, dot and dash lines, respectively. It is noticeable from equation (5) that if the depth of the potential well U exceeds the summation of E_p and $2mc^2$, the wavefunction in the potential well region will be evanescent, similar to the case of the tunnelling potential barrier [11]. This is completely different from the nonrelativistic case. The oscillation of Larmor time with respect to the potential well thickness displayed in figure 1 is related to the periodical occurrence of transmission resonances at $ka/\hbar = n\pi$ (with n a positive integer), at which Larmor time becomes $aE\left(k^2+f_0^2\xi^2\right)/2f_0c^2k^2\xi$. From the wave viewpoint, the oscillation is due to the quantum phase interference between the incoming wave and the reflected wave from the rear

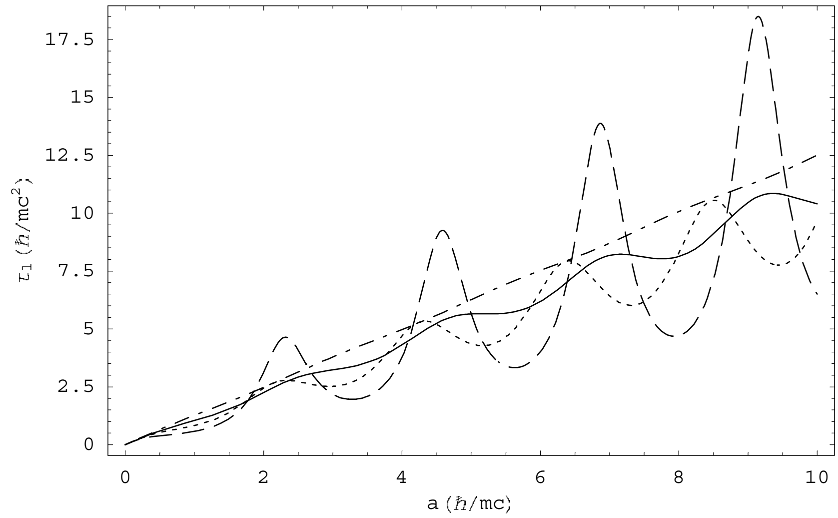

Figure 1. The dependence of Larmor time calculated from expression (21) with a potential well on the well width a, where $E_p = 0.5mc^2$ and $U = 0.1mc^2, 0.4mc^2, 0.8mc^2, 1.6mc^2$ corresponding to the dash-dot, solid, dot and dash lines, respectively.

edge of the potential well. The condition of constructive interference, which leads to the maximum probability of the wavefunction in the well and therefore the longest interaction time, is obviously $a = n\pi\hbar/k$. With increasing incident particle energy compared with the depth of the potential well, the influence of the potential well on the particle becomes weak, so the oscillations disappear and finally the Larmor time is proportional to the well thickness. The dependence of Larmor time on the thickness in the potential well case is very different from that in the potential barrier case [24], in which the Hartman effect is shown [14].

It is interesting to compare the Larmor time of a relativistic particle traversing a well with the Larmor time of a relativistic particle traversing a constant magnetic field **B** confined in region $-a/2 < x < a/2$, but without a well. With the same procedure as that for the case with a potential well, the Larmor time of passage through the magnetic field region in the absence of a well is

$$\tau_L^0 = \frac{aE}{c^2 k_0} \tag{22}$$

which is exactly the ratio of the travelling distance a to speed $v = c\sqrt{1-(mc^2/E)^2}$. Choosing $U = 0.8mc^2$ and $a = 3\hbar/mc$, figure 2 shows the dependence of Larmor time τ_L (solid line) and τ_L^0 (dash line) on the incident particle kinetic energy E_p in units of mc^2. It has been displayed obviously that Larmor time τ_L with a well is always smaller than Larmor time τ_L^0 without a well. In other words, the speed of a particle in a potential well is greater than that in free space, which is likely to show apparent superluminality. For the neutron $m = 1.67 \times 10^{27}$ kg, if we choose $a = 5 \times 10^6\hbar/mc$ (about 10 Å), $E = 1.5mc^2$ and $U = 0.8mc^2$, the speed of the particle through the potential well would be $1.1c$. The oscillation of Larmor time with the potential well is also related to the periodical occurrence of transmission resonances at $ka/\hbar = n\pi$ and due to the quantum phase interference between the incoming wave and the reflected wave from the rear edge of the potential well. The peaks correspond to the energy levels of the particle in the potential well, which increase with the depth of the potential well. With increasing incident particle energy, the influence of the potential well on the particle

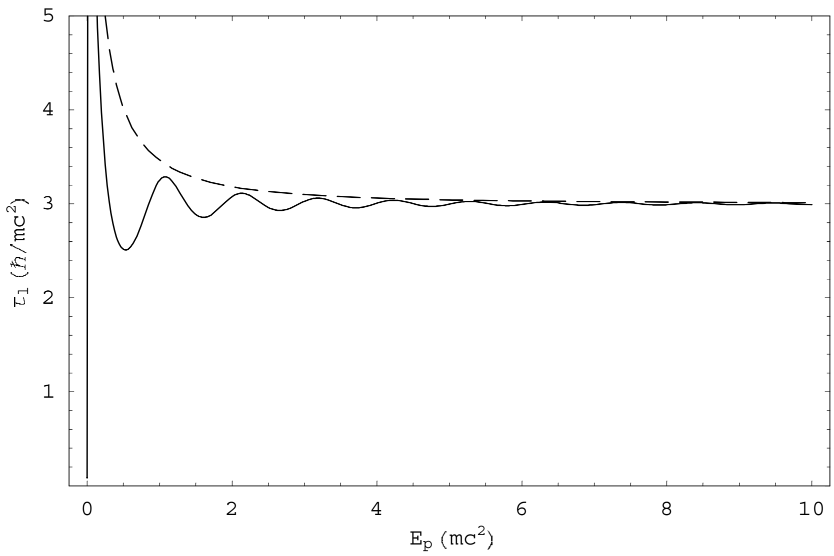

Figure 2. The dependence of Larmor time of expression (21) with a well (solid line) and Larmor time of expression (22) without a well (dash line) on the incident particle energy, where $U = 0.8mc^2$ and $a = 3\hbar/mc$.

becomes weak, so the Larmor time with a well asymptotically tends to be the Larmor time without a well.

The dwell time τ_d of a particle in a rectangular well is defined as the ratio of the probability P_b of finding a particle within the well to the incident probability flux [17],

$$\tau_d = \frac{P_b}{J_i}. \tag{23}$$

In Dirac theory, the incident probability flux J_i is

$$J_i = \psi_i^+ c\alpha_1 \psi_i = \frac{2cf_0}{1 + f_0^2}. \tag{24}$$

The probability for the particle to be in the well can be obtained from the determinable coefficients of equation (8) , which in infinitesimal field limit is

$$\begin{aligned} P_b &= \int_{-a/2}^{a/2} \psi_2^+ \psi_2 \, \mathrm{d}x \\ &= \frac{f_0^2\left[2ak(k^2+\xi^2)\left(k^2+f_0^2\xi^2\right) - \hbar(k^2-\xi^2)\left(k^2-f_0^2\xi^2\right)\sin\left(\frac{2ak}{\hbar}\right)\right]}{\left(1+f_0^2\right)k\left[4f_0^2k^2\xi^2+\left(k^2-f_0^2\xi^2\right)^2\sin^2\left(\frac{ak}{\hbar}\right)\right]}. \end{aligned} \tag{25}$$

The dwell time τ_d is

$$\tau_d = \frac{f_0}{2ck}\frac{2ak(k^2+\xi^2)\left(k^2+f_0^2\xi^2\right) - \hbar(k^2-\xi^2)\left(k^2-f_0^2\xi^2\right)\sin\left(\frac{2ak}{\hbar}\right)}{4f_0^2k^2\xi^2+\left(k^2-f_0^2\xi^2\right)^2\sin^2\left(\frac{2dk}{\hbar}\right)}. \tag{26}$$

Using the relation

$$E = \frac{c\xi^2 + ck^2}{2\xi} \tag{27}$$

it is obvious that the dwell time (26) equals exactly the Larmor time (21). So we can utilize the spin precession of the particle in a uniform constant magnetic field to measure the dwell time of the particle in a rectangular well exactly.

To summarize, using the spin coherent state of an incoming particle we have shown that a relativistic neutral spin-$\frac{1}{2}$ particle penetrating a rectangular potential well with a uniform constant magnetic field gives rise to a Larmor precession, from which the dwell time for the particle to remain in the well is determined . The Larmor time of the particle precession in a well oscillates with the well thickness and depth and asymptotically tends to the Larmor time in free space. Similar to the Larmor time of a relativistic particle tunnelling a rectangular barrier, the Larmor time of a relativistic particle in a well is smaller than that in free space, which implies apparent superluminality.

Acknowledgments

This work was supported by the Youth Science Foundation of Shanxi Province and the National Nature Science Foundation of China under grant no 10075032.

References

[1] Hauge E H and Støvneng J A 1989 *Rev. Mod. Phys.* **61** 917
[2] Landauer R and Martin T 1994 *Rev. Mod. Phys.* **66** 217
[3] Steinberg A M 1995 *Phys. Rev. Lett.* **74** 2405
[4] Wang L J, Kuzmich A and Dogariu A 2000 *Nature* **406** 277
[5] Mugnai D, Ranfagni A and Ruggeri R 2000 *Phys. Rev. Lett.* **84** 4830
[6] Steinberg A M and Chiao R Y 1994 *Phys. Rev.* A **49** 3283
[7] Li C F and Wang Q 2000 *Phys. Lett.* A **275** 287
[8] Wigner E P 1955 *Phys. Rev.* **98** 145
[9] Vetter R M, Haibel A and Nimtz G 2001 *Phys. Rev.* E **63** 046701
[10] Jackson A D, Lande A and Lautrup B 2001 *Phys. Rev.* A **64** 044101
[11] Li Z J, Liang J Q and Kobe D H 2001 *Phys. Rev.* A **64** 042112
[12] Li Z J, Liang J Q and Kobe D H 2002 *Phys. Rev.* A **65** 024101
[13] Liang J Q and Ding X X 1999 *Acta Phys. Sin.* **8** 409
[14] Li Z J, Nie Y H, Liang J Q and Yan Q W 2002 *Chin. Phys. Lett.* **19** 10
[15] Baz A I 1967 *Sov. J. Nucl. Phys.* **4** 182 (1966 *Yad. Fiz.* **4** 252 (in Russian))
[16] Rybachenko V F 1967 *Sov. J. Nucl. Phys.* **5** 635 (1966 *Yad. Fiz.* **5** 895 (in Russian))
[17] Büttiker M 1983 *Phys. Rev.* B **27** 6178
[18] Leavens C R and Aers G C 1987 *Solid State Commun.* **63** 1101
[19] Leavens C R and Aers G C 1988 *Solid State Commun.* **67** 1135
[20] Sokolovski D and Baskin L M 1987 *Phys. Rev.* A **36** 4604
[21] Gasparian V and Pollak M 1993 *Phys. Rev.* B **47** 2038
[22] Smith F T 1960 *Phys. Rev.* **118** 349
[23] Hartman T E 1962 *J. Appl. Phys.* **33** 3427
[24] Krekora P, Su Q and Grobe R 2001 *Phys. Rev.* A **63** 032107
[25] Hagen C R 1990 *Phys. Rev. Lett.* **64** 2347